In[1]:=
```
If[! TrueQ[Private`$ev], Block[{Private`$ev = True},
  Print["Total time: ", First@AbsoluteTiming[ClearAll["Global`*"];
     NotebookEvaluate[EvaluationNotebook[], InsertResults → True]], " s"]]]
```

```
Total time: 43.3488 s
```

In[2]:=
```
ClearAll["Global`*"];
(* functions in (A.2)*)
q1[t_] := 21 539 / 93 423 + 127 023 / 185 578 * t^2 - 1 / 8 885 055 * t^3 +
   54 169 / 401 949 * t^4 - 3 / 44 981 * t^5 + 202 / 73 305 * t^6 - 113 / 80 657 * t^7 +
   293 / 59 051 * t^8 - 604 / 151 861 * t^9 + 127 / 76 892 * t^10 - 28 / 94 431 * t^11;
q2[t_] := 18 176 / 78 783 - 21 / 15 850 t + 295 367 / 428 350 t^2 - 1415 / 123 249 t^3 +
   31 027 / 204 823 t^4 - 5162 / 329 873 t^5 + 3025 / 287 391 t^6 - 17 / 36 388 t^7 +
   2 / 74 523 t^8 + 5 / 86 563 t^9 - 1 / 120 831 t^10 + 1 / 1 183 575 t^11;
f1[t_] := Exp[-t] / t;
p2[t_] := q2[t] + q1[9 / 10] - q2[9 / 10] + (q1'[9 / 10] - q2'[9 / 10]) (t - 9 / 10);
ggg[t_] := (2 Exp[t] ExpIntegralEi[-4 t] - Exp[-t] ExpIntegralEi[-2 t]) / t;
AAAAAA = (p2[5 / 2] × ggg'[5 / 2] + p2'[5 / 2] × ggg[5 / 2]) /
    (f1[5 / 2] × ggg'[5 / 2] - f1'[5 / 2] × ggg[5 / 2]) 1 / (p2[5 / 2])^2;
BBBBBB = -(p2[5 / 2] × f1'[5 / 2] + p2'[5 / 2] × f1[5 / 2]) /
    (f1[5 / 2] × ggg'[5 / 2] - f1'[5 / 2] × ggg[5 / 2]) 1 / (p2[5 / 2])^2;
Q[t_] :=
  If[t ≤ 9 / 10, 1 / q1[t], If[t ≤ 5 / 2, 1 / p2[t], AAAAAA Exp[-t] / t + BBBBBB ggg[t]]];
ETA[t_] := 7 × 10^(-5) Exp[-t] / (1 + t);
(*Q_*^2 in (A.3)*)
Qanew[t_] := -32 389 / 844 * t^14 + 692 573 / 1409 * t^13 -
   14 647 839 / 5182 * t^12 + 13 634 891 / 1418 * t^11 - 31 307 644 / 1461 * t^10 +
   22 954 676 / 703 * t^9 - 1 268 792 / 37 * t^8 + 34 165 996 / 1415 * t^7 -
   12 679 168 / 1223 * t^6 + 2 124 876 / 1117 * t^5 + 78 128 / 353 * t^4 +
   15 317 / 829 * t^3 - 138 192 / 1231 * t^2 + 15 295 / 813;
(* W_1 in (A.4)*)
W1[t_] :=
  -126 751 t^22 / 80 621 266 + 998 813 t^21 / 24 102 701 - 44 784 703 t^20 / 90 944 085 +
   532 787 760 t^19 / 154 914 809 - 1 478 230 681 t^18 / 97 178 472 +
   1 104 148 971 t^17 / 27 114 439 - 6 386 998 649 t^16 / 165 889 120 -
   4 563 349 408 t^15 / 24 620 175 + 12 295 664 789 t^14 / 11 848 354 -
   25 742 069 395 t^13 / 8 929 817 + 22 711 315 187 t^12 / 4 190 364 -
   21 505 639 879 t^11 / 2 916 474 + 32 713 853 029 t^10 / 4 462 850 -
   12 946 708 519 t^9 / 2 496 198 + 3 403 022 733 t^8 / 1 411 424 -
   6 729 001 962 t^7 / 11 651 297 - 626 483 074 t^5 / 65 670 795 +
   752 248 559 t^4 / 20 846 520 - 89 936 485 t^2 / 6 984 383 + 93 423 / 21 539;
(* W_2 in (A.5)*)
W2[t_] := 157 * t^8 / 1 255 706 - 5215 * t^7 / 1 535 379 + 25 008 * t^6 / 610 441 -
   1 118 777 * t^5 / 3 886 821 + 1 230 788 * t^4 / 949 047 - 784 873 * t^3 / 202 656 +
   10 232 110 * t^2 / 1 348 117 - 2 084 185 * t / 228 972 + 43 251 / 8233;
(* P_1, P_2 in (B.2)*)
P1[t_] := 2 (W1[t] - 75 × 10^(-6))^2;
```

```
P2[t_] := (W1[t] + 75 × 10^(-6)) (W1'[t] - 42 × 10^(-5));
(* P3, P4 in (B.3)*)
P3[t_] := 2 (W1[t] + 75 × 10^(-6))^2;
P4[t_] := (W1[t] + 75 × 10^(-6))^4;
(*The following function polynomialMin
 corresponds to formula (1.25) in Lemma 1.4*)
polynomialMin[p_, a_, b_, dx_] := Module[{d, pValues, m1, m2, lam, Nsub, coeffList},
  If[! PolynomialQ[p, x], Return["Error: p must be a polynomial in x."];];
  coeffList = CoefficientList[p, x];
  d = Exponent[p, x];
  Nsub = Ceiling[(b - a) / dx];
  If[6 Nsub^2 - d^2 (d^2 - 1) ≤ 0, Return["Error: lambda must be positive."];];
  pValues =
   ParallelTable[FromDigits[Reverse[coeffList], a + k * dx], {k, 0, Nsub}];
  m1 = Min[pValues];
  m2 = Max[Abs[pValues]];
  lam = (d^2 (d^2 - 1)) / (6 Nsub^2 - d^2 (d^2 - 1));
  m1 - lam * m2]
```

Appendix A

(A .6)

In[18]:=
```
Clear[p]; p = Qanew[x];
polynomialMin[p, 0, 8 / 5, 10^(-4)] > 0
Clear[p]; p = -Simplify[Qanew'[x] / x];
polynomialMin[p, 0, 8 / 5, 10^(-4)] > 0
```

Out[19]= True

Out[21]= True

(A .8)

In[22]:=
```
(*Check the first inequality in (A.8) via Corollary 1.5*)
Clear[p];
p = 75 * 10^(-5) q1[x]^2 + (q1[x]^2 × Qanew[x] - (1 - 7 * 10^(-5) q1[x])^2);
polynomialMin[p, 0, 9 / 10, 10^(-4)] > 0
Clear[p];
p = 75 * 10^(-5) q1[x]^2 - (q1[x]^2 × Qanew[x] - (1 - 7 * 10^(-5) q1[x])^2);
polynomialMin[p, 0, 9 / 10, 10^(-4)] > 0
```

Out[23]= True

Out[25]= True

In[26]:=
```
(*Check the second inequality in (A.8) via Corollary 1.5*)
Clear[p];
p = 75 * 10^(-5) p2[x]^2 + (p2[x]^2 × Qanew[x] - (1 - 7 * 10^(-5) p2[x])^2);
polynomialMin[p, 9 / 10, 8 / 5, 10^(-4)] > 0
Clear[p];
p = 75 * 10^(-5) p2[x]^2 - (p2[x]^2 × Qanew[x] - (1 - 7 * 10^(-5) p2[x])^2);
polynomialMin[p, 9 / 10, 8 / 5, 10^(-4)] > 0
```

Out[27]= True

Out[29]= True

(A .9)

In[30]:=
```
(*Check the first inequality in (A.9) via Corollary 1.5*)
Clear[p];
p = 75 * 10^(-5) q1[x]^2 + (q1[x]^2 × Qanew[x] - (1 + 7 * 10^(-5) q1[x])^2);
polynomialMin[p, 0, 9 / 10, 10^(-4)] > 0
Clear[p];
p = 75 * 10^(-5) q1[x]^2 - (q1[x]^2 × Qanew[x] - (1 + 7 * 10^(-5) q1[x])^2);
polynomialMin[p, 0, 9 / 10, 10^(-4)] > 0
```

Out[31]= True

Out[33]= True

In[34]:=
```
(*Check the second inequality in (A.9) via Corollary 1.5*)
Clear[p];
p = 75 * 10^(-5) p2[x]^2 + (p2[x]^2 × Qanew[x] - (1 + 7 * 10^(-5) p2[x])^2);
polynomialMin[p, 9 / 10, 8 / 5, 10^(-4)] > 0
Clear[p];
p = 75 * 10^(-5) p2[x]^2 - (p2[x]^2 × Qanew[x] - (1 + 7 * 10^(-5) p2[x])^2);
polynomialMin[p, 9 / 10, 8 / 5, 10^(-4)] > 0
```

Out[35]= True

Out[37]= True

(A .10)

In[38]:=
```
(*Check the first inequality in (A.10) via Corollary 1.5*)
Clear[p]; p = 32 * 10^(-7) q1[x] + (1 - q1[x] × W1[x]);
polynomialMin[p, 0, 9 / 10, 10^(-4)] > 0
Clear[p]; p = 32 * 10^(-7) q1[x] - (1 - q1[x] × W1[x]);
polynomialMin[p, 0, 9 / 10, 10^(-4)] > 0
```

Out[39]= True

Out[41]= True

In[42]:=
```
(*Check the second inequality in (A .10) via Corollary 1.5*)
Clear[p]; p = 32 * 10^(-7) p2[x] + (1 - p2[x] × W1[x]);
polynomialMin[p, 9 / 10, 5 / 2, 10^(-4)] > 0
Clear[p]; p = 32 * 10^(-7) p2[x] - (1 - p2[x] × W1[x]);
polynomialMin[p, 9 / 10, 5 / 2, 10^(-4)] > 0
```

Out[43]= True

Out[45]= True

(A .11)

In[46]:=
```
(*Check the first inequality in (A.11) via Corollary 1.5*)
Clear[p]; p = 57 * 10^(-6) q1[x]^2 + (q1'[x] + q1[x]^2 × W1'[x]);
polynomialMin[p, 0, 9 / 10, 10^(-5)] > 0
Clear[p]; p = 57 * 10^(-6) q1[x]^2 - (q1'[x] + q1[x]^2 × W1'[x]);
polynomialMin[p, 0, 9 / 10, 10^(-5)] > 0
```

Out[47]= True

Out[49]= True

In[50]:=
```
(*Check the second inequality in (A.11) via Corollary 1.5*)
Clear[p]; p = 57 * 10^(-6) p2[x]^2 + (p2'[x] + p2[x]^2 × W1'[x]);
polynomialMin[p, 9 / 10, 5 / 2, 10^(-4)] > 0
Clear[p]; p = 57 * 10^(-6) p2[x]^2 - (p2'[x] + p2[x]^2 × W1'[x]);
polynomialMin[p, 9 / 10, 5 / 2, 10^(-4)] > 0
```

Out[51]= True

Out[53]= True

(A .13)

In[54]:=
```
(*Check the first inequality in (A.13) via Corollary 1.5*)
Clear[p]; p = 18 * 10^(-7) * p2[x] + (1 - p2[x] × W1[x]);
polynomialMin[p, 18 / 10, 5 / 2, 10^(-4)] > 0
Clear[p]; p = 18 * 10^(-7) * p2[x] - (1 - p2[x] × W1[x]);
polynomialMin[p, 18 / 10, 5 / 2, 10^(-4)] > 0
```

Out[55]= True

Out[57]= True

In[58]:=
```
(*Check the second inequality in (A.13) via Corollary 1.5*)
Clear[p]; p = 29 * 10^(-6) * p2[x]^2 + (p2'[x] + p2[x]^2 × W1'[x]);
polynomialMin[p, 18 / 10, 5 / 2, 10^(-4)] > 0
Clear[p]; p = 29 * 10^(-6) * p2[x]^2 - (p2'[x] + p2[x]^2 × W1'[x]);
polynomialMin[p, 18 / 10, 5 / 2, 10^(-4)] > 0
```

Out[59]= True

Out[61]= True

(A .14)

In[62]:=
```
Clear[p];
p = 48 * 10^(-7) * x + (x W2''[x] + 2 W2'[x] - x W2[x] + x W2[x]^3);
polynomialMin[p, 5 / 2, 7 / 2, 10^(-4)] > 0
Clear[p];
p = 48 * 10^(-7) * x - (x W2''[x] + 2 W2'[x] - x W2[x] + x W2[x]^3);
polynomialMin[p, 5 / 2, 7 / 2, 10^(-4)] > 0
```

Out[63]= True

Out[65]= True

(A .15)

In[66]:=
```
polynomialMin[W2[x], 5 / 2, 7 / 2, 10^(-4)] > 0
polynomialMin[9 / 100 - W2[x], 5 / 2, 7 / 2, 10^(-4)] > 0
```

Out[66]= True

Out[67]= True

(A.18)

In[68]:=
$$\frac{48 \times 10^{-7}}{\left(1 - 2\,\frac{243}{10\,000} - \frac{81}{10\,000}\right)} < \frac{51}{10^7}$$

Out[68]= True

$$\max_{\frac{5}{2} \le t \le \frac{7}{2}} \left| \eta(t) \right| < \frac{5}{10^6}$$

In[69]:=
$$\frac{3 \times \frac{9}{100}\left(\frac{51}{10^7}\right)^2 + \left(\frac{51}{10^7}\right)^3 + 48 \times 10^{-7}}{1 - 3\left(\frac{243}{10\,000}\right)^2} < \frac{5}{10^6}$$

Out[69]= True

(A.19)

In[70]:=
$$\frac{48 \times 10^{-7}}{1 - \frac{243}{10\,000} - \frac{81}{10\,000}} < \frac{5}{10^6}$$

Out[70]= True

(A.20)

In[71]:=
$$\frac{83}{10^7} + \frac{109}{147}\left(\frac{5}{10^6} + \frac{48}{10^7}\right) \leq \frac{156}{10^7}$$

Out[71]= True

footnote 38

In[72]:=
$$\left(\frac{7}{2}\right)^3 \left(W2'\left[\frac{7}{2}\right] - \frac{156}{10^7}\right) - \left(\frac{7}{2}\right)^2 \left(W2\left[\frac{7}{2}\right] + \frac{5}{10^6}\right) > -2$$

Out[72]= True

(A.21)

In[73]:=
$$\left\{\left(\frac{17}{10}\right)^3 \left(W1'\left[\frac{17}{10}\right] - \frac{42}{10^5}\right) - \left(\frac{17}{10}\right)^2 \left(W1\left[\frac{17}{10}\right] + \frac{75}{10^6}\right) > -\frac{31}{10},\right.$$
$$\left.\left(\frac{17}{10}\right)^2 \left(1 - \left(W1\left[\frac{17}{10}\right] + \frac{75}{10^6}\right)^2\right) - 2 > 0\right\}$$

$$W1\left[\frac{17}{10}\right] + \frac{75}{10^6} < \frac{\frac{93}{10} - \sqrt{\left(-\frac{93}{10}\right)^2 - 4 \times \frac{867}{100} \times 2}}{2 \times \frac{867}{100}}$$

Out[73]= {True, True}

Out[74]= True

(A.22)

In[75]:=
$$2 - 6\left(W2\left[\frac{7}{2}\right] + 156 \times 10\,\text{^}\,(-7)\right) > \frac{18\,525}{10\,000}$$

Out[75]= True

(A .23)

In[76]:=
```
Clear[p]; p = 2 + x^3 (W1[x] + 75 * 10^(-6)) (W1'[x] - 42 * 10^(-5));
polynomialMin[p, 0, 17 / 10, 10^(-4)] > 0
```

Out[77]= True

Appendix B

Case 1≤τ≤1.2

(B.4)

In[78]:=
```
Clear[A]; A = 3 / 10^4; 1 + 10 / 13 (-12872 / 10000) > 98 / 10000
(31 / 100 - 2 (W2[7 / 2] + 5 × 10^(-6))^2 - 1 / (2 A (26 / 10)^2) (W2[7 / 2] + 5 × 10^(-6))^4) -
 A / 4 (W2[7 / 2] + 5 × 10^(-6))^2 > 0
```

Out[78]= True

Out[79]= True

(B.5)

In[80]:=
```
Clear[A];
A = 3 / 10^4;
31 / 100 - 2 (W1[5 / 2] + 75 × 10^(-6))^2 - 1 / (2 A (26 / 10)^2) (W1[5 / 2] + 75 × 10^(-6))^4 > 27 / 100
```

Out[80]= True

(B.6)

In[81]:=
```
Clear[af, p];
af = 1 / 2441 Sum[1 / k! (-13 / 5 (x - 3)) ^k, {k, 0, 3}] ;
p =
  W2[x] - 5 × 10^(-6) + 2 (10 / 26 - 630 / 26 af) (W2'[x] - 156 × 10^(-7)) - 2 × 10^(-7);
polynomialMin[p, 5 / 2, 7 / 2, 10^(-3)] > 0
```

Out[84]= True

$\frac{1}{2} AQ\left(\frac{8}{5}\right)^4 < \frac{3}{10^6}$ before (B.9)

In[85]:=
$$\text{Clear[A]; A} = \frac{3}{10^4}; \frac{1}{2}\, A \left(W1\left[\frac{8}{5}\right] + 75 \times 10\text{^}(-6)\right)\text{^}4 < \frac{3}{10^6}$$

Out[85]= True

(B.9)

In[86]:=
```
Clear[a1f, p, A, pnew];
a1f = 73 / 15 070 Sum[1 / k! (-13 / 5 (x - 41 / 20)) ^k, {k, 0, 5}];
p = P1[x] + (10 / 26 - 630 / 26 a1f) 4 P2[x];
A = 3 × 10^(-4);
pnew = 31 / 100 - P3[x] - 1 / (2 A) (10 / 26 - 630 / 26 a1f) ^2 P4[x];
{polynomialMin[p, 8 / 5, 5 / 2, 15 / 10^6] > 5 / 2 × 10^(-4),
 polynomialMin[pnew, 8 / 5, 5 / 2, 2 / 10^5] > 2 × 10^(-2)}
```

Out[91]= {True, True}

Case $1.2 \leq \tau \leq 2.1$

(B.10)

In[92]:=
$$\text{Clear[A]; A} = \frac{2}{10^3};$$
$$\left\{\frac{1}{2}\, A \left(W1\left[\frac{5}{2}\right] + 75 \times 10\text{^}(-6)\right)\text{^}4 < \frac{1}{10^4},\right.$$
$$\frac{2}{10} + \left(2 - \frac{40}{14} \times \frac{380}{271}\right) \left(W1\left[\frac{5}{2}\right] + 75 \times 10\text{^}(-6)\right)\text{^}2 > \frac{1}{10},$$
$$\left.\frac{171}{100} - 2 \left(W1\left[\frac{5}{2}\right] + 75 \times 10\text{^}(-6)\right)\text{^}2 - \frac{1}{2\, A} \left(\frac{10}{14}\right)\text{^}2 \left(W1\left[\frac{5}{2}\right] + 75 \times 10\text{^}(-6)\right)\text{^}4 > 1\right\}$$

Out[93]= {True, True, True}

$\frac{1}{2}AQ\left(\frac{11}{10}\right)^4<\frac{1}{10^3}$ before (B.11)

In[94]:=
```
Clear[A]; A = 2/10^3; 1/2 A (W1[11/10] + 75 × 10^(-6))^4 < 1/10^3
```
Out[94]= True

(B.11)

In[95]:=
```
Clear[p, pnew, a2f, x, A];
a2f = 8 / 100 Sum[1 / k! (-7 / 5 (x - 9 / 5))^k , {k, 0, 3}]; A = 2 * 10^(-3);
p = 1 / 5 + P1[x] + (10 / 14 - 461 / 140 a2f) 4 P2[x];
pnew = 171 / 100 - P3[x] - 1 / (2 A) (10 / 14 - 461 / 140 a2f)^2 P4[x];
{polynomialMin[p, 11 / 10, 5 / 2, 2 / 10^4] > 1 / 10,
 polynomialMin[pnew, 11 / 10, 5 / 2, 1 / 10^4] > 2 / 100}
```
Out[98]= {True, True}

Case $2.1 \le \tau \le 5.4$

(B.12)

In[99]:=
```
Clear[A]; A = 1/100;
{1/2 A (W1[5/2] + 75 × 10^(-6))^4 < 1/10^4,
 11/10 + (2 - 4 × 380/271 × 5/2) (W1[5/2] + 75 × 10^(-6))^2 > 1,
 31/10 - 2 (W1[5/2] + 75 × 10^(-6))^2 - 1 / (2 A) (5 / 2)^2 (W1[5/2] + 75 × 10^(-6))^4 > 1}
```
Out[100]= {True, True, True}

$\frac{1}{2}AQ\left(\frac{91}{100}\right)^4<\frac{9}{10^3}$ and $\frac{1}{2}AQ(T)^4<\frac{12}{10^3}$ before (B.14)

In[101]:=
```
Clear[A, T];
A = 1/100;
T = 856/1000;
{1/2 A (W1[91/100] + 75 × 10^(-6))^4 < 9/10^3, 1/2 A (W1[T] + 75 × 10^(-6))^4 < 12/10^3}
```
Out[101]= {True, True}

(B.14) and (B.15)

In[102]:=
```
Clear[p, pnew];
p = 11 / 10 + P1[x] + 4 (x - 845 / 1000) P2[x];
A = 10^(-2);
pnew = 31 / 10 - P3[x] - 1 / (2 A) (x - 845 / 1000)^2 P4[x];
 {polynomialMin[p, 91 / 100, 5 / 2, 15 / 10^5] > 7 / 10,
  polynomialMin[pnew, 91 / 100, 5 / 2, 8 / 10^5] > 16 / 100}
{polynomialMin[p, 856 / 1000, 91 / 100, 2 / 10^6] > 3,
  polynomialMin[pnew, 856 / 1000, 91 / 100, 9 / 10^7] > 41 * 10^(-4)}
```

Out[104]= {True, True}

Out[105]= {True, True}

Case $5.4 \le \tau \le 12.4$

$$\frac{1}{2} AQ\left(\frac{5}{2}\right)^4 < \frac{1}{10^4}$$

$$f_1 > \frac{44}{10} + 2\,Q\left(\frac{5}{2}\right)^2 + 4 \cdot \left(-\frac{380}{271}\right) \cdot \frac{5}{2} \cdot Q\left(\frac{5}{2}\right)^2 > 1, \quad \forall\, t \ge \frac{5}{2};$$

$$f_2 > \frac{64}{10} - 2\,Q\left(\frac{5}{2}\right)^2 - \frac{1}{2A} \cdot \left(\frac{5}{2}\right)^2 \cdot Q\left(\frac{5}{2}\right)^4 > 1, \quad \forall\, t \ge \frac{5}{2}.$$

In[106]:=
```
Clear[A]; A = 2/100;
{1/2 A (W1[5/2] + 75 × 10^(-6))^4 < 1/10^4,
 44/10 + (2 - 4 × 380/271 × 5/2) (W1[5/2] + 75 × 10^(-6))^2 > 1,
 64/10 - 2 (W1[5/2] + 75 × 10^(-6))^2 - 1 / (2 A) (5 / 2)^2 (W1[5/2] + 75 × 10^(-6))^4 > 1}
```

Out[107]= {True, True, True}

$\frac{1}{2}AQ\left(\frac{68}{100}\right)^4 < \frac{1}{10}$ and $\frac{1}{2}AQ(T)^4 < \frac{12}{100}$ before (B.16)

In[108]:=
```
Clear[A, T];
A = 2/100;
T = 2/3;
{1/2 A (W1[68/100] + 75 × 10^(-6))^4 < 1/10, 1/2 A (W1[T] + 75 × 10^(-6))^4 < 12/100}
```

Out[108]= {True, True}

(B.16) and (B.17)

In[109]:=
```
Clear[p, pnew, A];
A = 2 * 10^(-2);
p = 44 / 10 + P1[x] + 4 (x - 655 / 1000) P2[x];
pnew = 64 / 10 - P3[x] - 1 / (2 A) (x - 655 / 1000)^2 P4[x];
{polynomialMin[p, 68 / 100, 5 / 2, 52 / 10^5] > 3,
 polynomialMin[pnew, 68 / 100, 5 / 2, 8 / 10^5] > 8 / 100}
{polynomialMin[p, 2 / 3, 68 / 100, 1 / (9 × 10^5)] > 98 / 10,
 polynomialMin[pnew, 2 / 3, 68 / 100, 1 / (12 × 10^5)] > 14 * 10^(-3)}
```

Out[111]= {True, True}

Out[112]= {True, True}

Case $12.4 \le \tau \le 19.2$

$$\frac{1}{2} A Q \left(\frac{5}{2}\right)^4 < \frac{1}{10^4}$$

$$f_1 > \frac{114}{10} + 2\,Q\left(\frac{5}{2}\right)^2 + 4 \cdot \left(-\frac{380}{271}\right) \cdot \frac{5}{2} \cdot Q\left(\frac{5}{2}\right)^2 > 1, \quad \forall\, t \ge \frac{5}{2};$$

$$f_2 > \frac{134}{10} - 2\,Q\left(\frac{5}{2}\right)^2 - \frac{1}{2A} \cdot \left(\frac{5}{2}\right)^2 \cdot Q\left(\frac{5}{2}\right)^4 > 1, \quad \forall\, t \ge \frac{5}{2}.$$

In[113]:=
```
Clear[A]; A = 32 / 1000;
{1/2 A (W1[5/2] + 75 × 10^(-6))^4 < 1/10^4,
 114/10 + (2 - 4 × 380/271 × 5/2) (W1[5/2] + 75 × 10^(-6))^2 > 1,
 134/10 - 2 (W1[5/2] + 75 × 10^(-6))^2 - 1 / (2 A) (5 / 2)^2 (W1[5/2] + 75 × 10^(-6))^4 > 1}
```

Out[114]= {True, True, True}

$\frac{1}{2} A Q\left(\frac{68}{100}\right)^4 < \frac{2}{10}$ and $\frac{1}{2} A Q(T)^4 < \frac{68}{100}$ before (B.18)

In[115]:=
```
Clear[A, T];
A = 32 / 1000;
T = 475 / 1000;
{1/2 A (W1[68/100] + 75 × 10^(-6))^4 < 2/10, 1/2 A (W1[T] + 75 × 10^(-6))^4 < 68/100}
```

Out[115]= {True, True}

(B.18) and (B.19)

In[116]:=
```
Clear[p, pnew, A];
A = 32 / 10^3;
p = 114 / 10 + P1[x] + 4 (x - 46 / 100) P2[x];
pnew = 134 / 10 - P3[x] - 1 / (2 A) (x - 46 / 100)^2 P4[x];
{polynomialMin[p, 68 / 100, 5 / 2, 2 / 10^4] > 95 / 10,
 polynomialMin[pnew, 68 / 100, 5 / 2, 1 / 10^4] > 5 / 100}
{polynomialMin[p, 475 / 1000, 68 / 100, 1 / 10^5] > 123 / 10,
 polynomialMin[pnew, 475 / 1000, 68 / 100, 1 / 10^5] > 59 / 1000}
```

Out[118]= {True, True}

Out[119]= {True, True}